# Gaussian Mixture Models for On-line Signature Verification

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## ABSTRACT

This paper introduces and motivates the use of Gaussian Mixture Models (GMMs) for on-line signature verification. The individual Gaussian components are shown to represent some local, signer-dependent features that characterise spatial and temporal aspects of a signature, and are effective for modelling its specificity. The focus of this work is on automated order selection for signature models, based on the Minimum Description Length (MDL) principle. A complete experimental evaluation of the Gaussian Mixture signature models is conducted on a 50-user subset of the MCYT multimodal database. Algorithmic issues are explored and comparisons to other commonly used on-line signature modelling techniques based on Hidden Markov Models (HMMs) are made.

## Categories and Subject Descriptors

K.6.5 [**Management of Computing and Information Systems**]: Security and Protection—*Authentication*; I.5.1 [**Pattern recognition**]: Models—*statistical*

## General Terms

Algorithms, Security, Verification, Experimentation

## Keywords

biometrics, signature verification, on-line signature, Gaussian mixture models, hidden Markov models, model order

## 1. INTRODUCTION

With the rise of e-commerce and remote-access banking, there has been a lot of interest in authentication solutions for securing transactions. Knowledge-based or physical-based access tokens suffer from various shortcomings as they can be forgotten, stolen, or duplicated. Biometrics-based access tokens, on the other hand, promise easier interactions and higher levels of security to the end-user.


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Biometric technologies can rely on two categories of traits: physiological traits (such as fingerprints, face, or iris), which the user is not required to perform any specific activity to produce, and behavioural traits (such as speech or handwritten signature), where the user is asked to perform a specific action. Handwritten signature, as a behavioural biometric, is socially acceptable, and is already commonly used in off-line, non-electronic forms. Furthermore, as mobile devices such as smartphones, PDAs and tablet PCs become more prevalent, the number of platforms with built-in support for pen input can be expected to rise and signature-based biometrics to become more commonplace, spreading applications such as electronic document authentication.

The signature verification task can be defined as follows: given a complete on-line signature $\boldsymbol{O}$ and a claimed user $u$, decide whether $u$ indeed produced the signature $\boldsymbol{O}$. Given a model $\boldsymbol{\Theta}^u$ for the claimed user and $\boldsymbol{\Theta}^-$ an antithetical model, a score function $S(\boldsymbol{O}, \boldsymbol{\Theta}^u, \boldsymbol{\Theta}^-)$, which for statistical models is generally a likelihood ratio, is used to determine whether the score of the input signature is above or below some threshold $\mathcal{T}$:

$$S(\boldsymbol{O}, \boldsymbol{\Theta}^u, \boldsymbol{\Theta}^-) \begin{cases} \geq \mathcal{T} & \text{accept identity claim} \\ < \mathcal{T} & \text{reject identity claim} \end{cases} \quad (1)$$

Over the past 15 years, a lot of research concerning on- and off-line signature verification has taken place. Numerous methods such as Dynamic Time Warping [6], hidden Markov models ([21], [3]), fuzzy logic inference [8], neural networks [19] have been used for on-line signature verification. Currently hidden Markov models (HMMs) exhibit state-of-the-art performance.

A statistical model which to the best of our knowledge has not been used in signature verification is the Gaussian mixture model (GMM). It is often used in verification-type systems, for instance in speaker verification [13] and face authentication [16]. A GMM can be thought of as a single-state hidden Markov model, which means HMM training methods can be applied to GMMs with very little or no modification. Their good performance in other applications of pattern recognition, combined with a research trend towards smaller number of states in HMM-based systems for on-line signature verification (20 states in [7], between 6 and 12 in [5], 5 in [22], 2 in [4]) leads us to think that they could be applied successfully to signature verification.

The rest of this paper is structured as follows: Section 2 explains the features extracted from the data and the structure of the signatures database used. Section 3 goes into the details of training GMMs and using them for signature verification, and evaluates the system's performance on this task. Section 4 compares GMM and HMM properties for signature verification.

## 2. SIGNATURE DATA AND FEATURES

The signatures used in our system are sampled at 100 Hz using a Wacom Intuos A6 tablet on which a paper alignment grid is placed, and each sample point (raw data vector) consists of values for the horizontal ($x$) position, vertical ($y$) position, pressure ($p$), azimuth, and elevation of the signing pen. Movement during pen-up ($p = 0$) is also recorded and used as part of the signature data. In this paper, we restrict the investigation to $x$, $y$, $p$ data because not all pen tablets can measure azimuth and elevation, whereas pressure-sensitivity is widespread.

This data was provided by the *Area de Tratamiento de Voz y Señales* (ATVS) of the Universidad Politécnica de Madrid, which manages a subcorpus of on-line signatures comprising authentic signatures and forgeries for 50 users, a subset of the larger MCYT multimodal database [10]. The data was acquired by requiring each user to produce 5 authentic signatures, then 5 skilled forgeries of another user's signature, and so on until a total of 25 authentic signatures and 25 skilled forgeries (of 5 different users) have been acquired. Skilled forgeries are obtained by giving forgers time to practise the signature they have to imitate before saving their attempt in the database.

Because of the acquisition methodology and inherent variability in initial pen-down position, it is necessary to make the $x$ and $y$ values translation-independent. This is achieved simply by subtracting the initial $x$ and $y$ values from all subsequent sample points. Rotation and scale differences are limited in the corpus used, because the strict acquisition grid, consisting of 3.3 cm x 1.2 cm boxes, constrains both signature orientation and size. Therefore, no transformations are used to counteract the effects of rotation or scaling.

Dynamic features are known to have good discriminative potential, and are difficult to reproduce from visual inspection of a paper realisation of the signature [7]. In addition to the $x$, $y$ positions and pressure values, it was decided to make use of trajectory tangent angles $\theta_t$ and instantaneous displacements (velocities) $v_t$, which are computed as

$$\theta_t = \arctan \frac{\dot{y}_t}{\dot{x}_t}, \quad v_t = \sqrt{\dot{x}_t^2 + \dot{y}_t^2} \tag{2}$$

where $\dot{y}_t$, $\dot{x}_t$ indicate first derivatives of $y_t$ and $x_t$ with respect to time. This results in a basis feature vector for each sample:

$$\tilde{\boldsymbol{o}}_t = [\ x_t, \quad y_t, \quad p_t, \quad \theta_t, \quad v_t\ ]' \tag{3}$$

An example for a complete on-line signature with these basis $D$=5 features is given in Figure 1.

Because the dynamic ranges of the different features varies widely, each individual feature $\tilde{o}_{dt}$ where $d = 1, \ldots, D$ is conformed to a zero-mean, unit variance normal distribution using:

$$o_{dt} = \frac{\tilde{o}_{dt} - \mu_{\tilde{o}_d}}{\sigma_{\tilde{o}_d}} \tag{4}$$

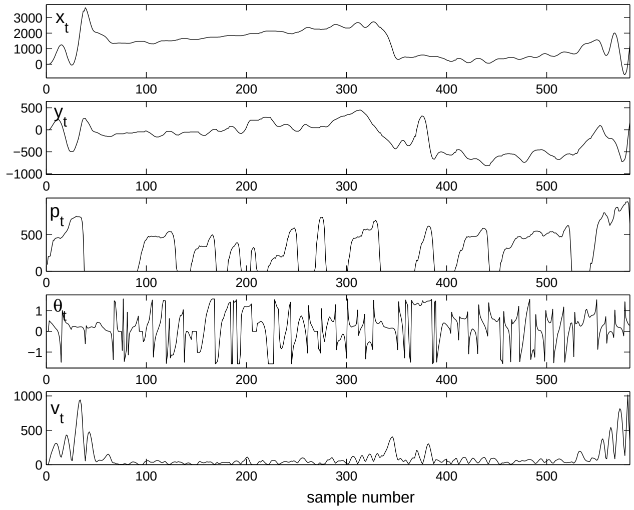


**Figure 1: Basis feature data $\tilde{O}$ for a signature**

The final complete observation $\boldsymbol{O} = [\boldsymbol{o}_1, \ldots, \boldsymbol{o}_t, \ldots, \boldsymbol{o}_T]$ is then obtained.

It should be noted however that this may not be a good choice for the pressure information, which is clearly multimodal in nature, as shown in Figure 2: pen-ups (null pressure) and maximum sensor pressure values are over-represented in the pressure value distribution. A feature normalisation technique such as feature warping [12] may be more appropriate.

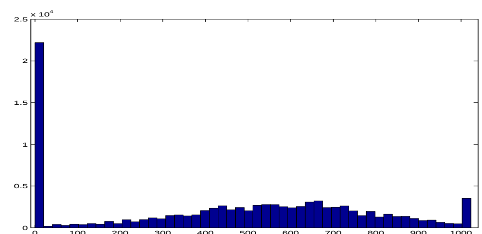

**Figure 2: 50-bins histogram of basis pressure values for 250 authentic signatures; note the large amount of null and sensor maximum pressure values**

## 3. GAUSSIAN MIXTURE MODELS FOR SIGNATURE RECOGNITION

GMMs are widely-used statistical models in many pattern recognition applications. They are a versatile modelling tool as they can be used to approximate any probability density function (pdf) given a sufficient number of components, and impose only minimal assumptions about the modelled random variables. With a $D$-dimensional feature vector $\mathbf{o}_t$ part of a complete observation sequence $\boldsymbol{O} = [\boldsymbol{o}_1, \ldots, \boldsymbol{o}_t, \ldots, \boldsymbol{o}_T]$, the general form of a probability density for an $M$-Gaussian pdf components GMM $\boldsymbol{\Theta}_M$ is:

$$p(\mathbf{o}_t|\boldsymbol{\Theta}_M) = \sum_{m=1}^{M} c_m \frac{e^{-\frac{1}{2}(\mathbf{o}_t-\boldsymbol{\mu}_m)'\boldsymbol{\Sigma}_m^{-1}(\mathbf{o}_t-\boldsymbol{\mu}_m)}}{|\boldsymbol{\Sigma}_m|^{\frac{1}{2}}(2\pi)^{\frac{D}{2}}} \tag{5}$$

Where $c_m$ is the Gaussian component weight (prior) with the constraint that $\sum_{m=1}^{M} c_m = 1$, $\boldsymbol{\mu}_m$ is the component mean, and $\boldsymbol{\Sigma}_m$ is the component's covariance matrix. If the elements in the feature vector are uncorrelated (or assumed to be), the covariance matrix becomes diagonal and Equation 5 can be simplified to:

$$p(\mathbf{o}_t|\boldsymbol{\Theta}_M) = \sum_{m=1}^{M} c_m \prod_{d=1}^{D} \frac{e^{-\frac{1}{2}\frac{(o_{dt}-\mu_{md})^2}{2\sigma_{md}^2}}}{\sqrt{2\pi\sigma_{md}^2}} \tag{6}$$

Using diagonal covariance matrices reduces the number of free parameters $N(M)$ in the model, from

$$N(M) = (M-1) + M\big(D + D(D+1)/2\big) \tag{7}$$

to

$$N(M) = M - 1 + 2MD \tag{8}$$

Also, diagonal covariance matrices reduce the number of operations for likelihood computations. However, if some degree of correlation exists between the features, as is often the case, the number of Gaussian components will need to be increased to account for it.

An overview of the GMM-based signature verification algorithm is provided in Figure 3.

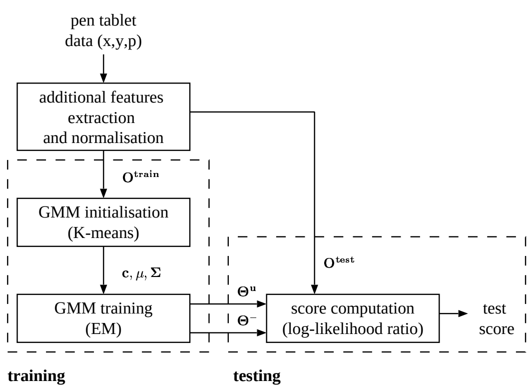


**Figure 3: GMM-based signature verification algorithm**

## 3.1 Model initialisation and training

The training data consisted of the first 5 authentic signatures for each user in the database. Because the time interval between acquisitions of the signature realisations was small, they are expected to exhibit lower variability than if the users had been asked to produce training signatures over a longer period of time; this may be detrimental to system performance as the intra-personal signature variability can be expected to augment with time. However, for real applications this acquisition scenario corresponds to established practice and represent a short and convenient enrolment procedure for users.

To achieve fast convergence, the model means are initialised using the K-means algorithm with randomly initialised and non-equal initial cluster centres. Then, the component weights, variances and means are iteratively re-estimated using 10 passes of the Expectation-Maximisation algorithm (EM), after which convergence is assumed.

## 3.2 Model order selection

Based on the system performance in terms of error rate for the chosen set of features, it was found that the optimal number of Gaussian components in the mixture is significantly smaller than that used in speaker verification tasks, where background models are often trained using 512 or 1024 components with diagonal covariance matrices. This is imputable to the fact that speaker verification systems typically use Mel-Frequency Cepstral Coefficients (MFCC) or Perceptual Linear Prediction (PLP) coefficients, which only approximately decorrelate the speech features, whereas the signature features presented in Section 2 are only weakly correlated. The correlation maps for PLP coefficients of a phone-quality conversation and the features of 25 realisations of a user's signature are shown in Figure 4.

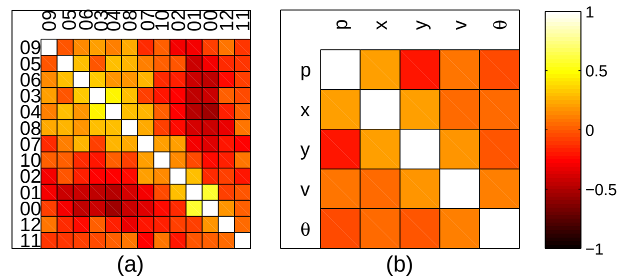


**Figure 4: Correlation maps for PLP coefficients 0-12 of a telephone conversation (a) and signature features of 25 realisations of the same signature (b). The scale indicates the relative value of the correlation coefficient $R$**

As with any pattern recognition system, the model order should neither be so low that the likelihood of observing the training data is very small, neither be so high that the model looses the capacity to predict unseen values of the modelled random variables. In practical cases, the likelihood of the training data given the model increases with the number of model parameters, so the likelihood function in itself is not a good indicator of overfitting. In [1] it has been suggested to use the knee in the curve of the *increase* of the log-likelihood to determine the optimal number of mixtures. However, this approach does not explicitly penalise more complex models.

The Minimum Description Length (MDL) principle [14] can be used to obtain a cost function that balances modelling errors and model complexity. Given a set of trained model parameters for different model orders, the number of training samples and the number of free parameters in the model, the minimum of the MDL cost function indicates the model that can represent both the data and the model parameters in the most compact fashion. For an $M$-components mixture model, an approximate expression for the MDL cost function can be written as

$$\mathrm{MDL}(\boldsymbol{\Theta}_M, M) = -\log p(\mathbf{O}|\boldsymbol{\Theta}_M) + \frac{1}{2}N(M)\log T \tag{9}$$

The first term will be small if the model fits the data well, thus reducing modelling errors. The second term will be large if the model has a large number of parameters, thus penalising complex models. The MDL criterion has been shown to be a consistent estimator of GMM order for a variety of problems [15].

Training four different full-covariance matrix GMMs with 8, 16, 24 and 32 Gaussian mixture components respectively showed that $\mathbf{\Theta}_{16}$ had slightly higher minimum description length than $\mathbf{\Theta}_{32}$, but that both 8- and 16-components models had significantly higher MDL values. The results are shown in Figure 5. This suggests that using either 16 or 32 full-covariance components per GMM will result in the most appropriate user signature models.

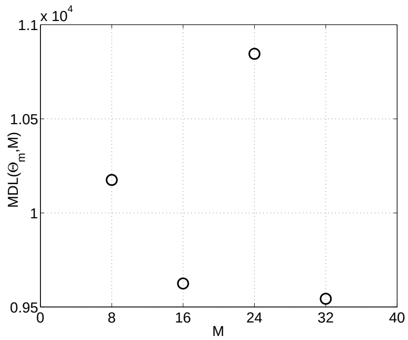


**Figure 5: Average MDL values for all users in the corpus with models using 8, 16, 24 and 32 full-covariance matrix Gaussian components**

Tests on diagonal-covariance matrix GMMs with 16, 32 and 64 components showed that the MDL criterion tended to under-estimate the optimal number of components; it was not found to be a good predictor of optimal model order for classification performance in the diagonal covariance case. This is in line with the findings in [15]. As a result, for diagonal covariance matrices the number of Gaussians was chosen with respect to the lowest Equal Error Rate (EER) obtained for the verification task described in Section 3.4.

## 3.3 Signature score computation and normalisation

A signature score is obtained as a ratio of the likelihood that observation $\boldsymbol{O}$ is seen given the model for user $u$ to the likelihood that any other user produced test signature $\boldsymbol{O}$. In other words, the score $S(\boldsymbol{O}, \mathbf{\Theta}^u, \mathbf{\Theta}^-)$ shows how different the test signature is from any other signature in the system world. It is computed as follows:

$$S(\boldsymbol{O}, \mathbf{\Theta}^u, \mathbf{\Theta}^-) = \log p(\boldsymbol{O}|\mathbf{\Theta}^u) - \log p(\boldsymbol{O}|\mathbf{\Theta}^{world}) \qquad (10)$$

Where $\mathbf{\Theta}^u$ is the model for user $u$ and $\mathbf{\Theta}^- = \mathbf{\Theta}^{world}$ is the world model. In the system proposed in this paper the world model is obtained by pooling together all enrolment data that the system has available (5 authentic signatures per user) and 5 same-forger forgery attempts per user. The world model has the same order as the user models. The resulting score is not aligned between users, which is equivalent to using a global threshold. Cohort models [11] and score alignment [4] could be used to improve performance.

The decision to accept or reject an identity claim can then be made by comparing the input score to a pre-determined threshold. An estimate for a user-independent threshold can be obtained by assuming that the distribution of scores is Gaussian and choosing the crossing point between the scores for forged signatures and authentic signatures.

## 3.4 Performance evaluation

The performance of the proposed GMM-based signature verification system was tested by scoring each user model against 20 authentic signatures and 25 skilled forgeries (from 5 different forgers), making a total test set of 1000 authentic signatures and 1250 forgeries. The results for diagonal covariance matrix models of different orders are presented in Figure 6 as a DET curve. An EER of 3.5% is obtained using 64 Gaussian components.

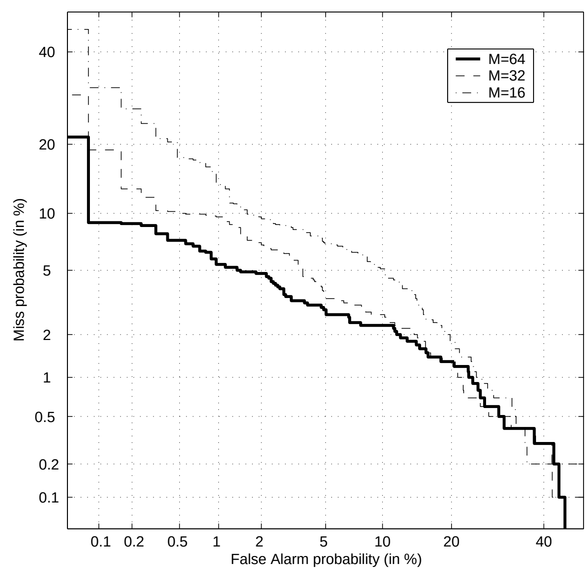


**Figure 6: DET plot for signature verification task with skilled forgeries using 16-, 32-, and 64-components GMM**

These results are of the same order as published HMM-based systems, demonstrating the effectiveness of Gaussian mixture models for signature verification tasks. The modelling could be further optimised by adding other features; additionally the pre-processing, number of training samples, training strategy, score normalisation, and score alignment could be modified to further improve results.

As an example of possible optimisation, the results of augmenting feature vectors of the baseline, diagonal-covariance 64-components GMM system with delta and delta-delta coefficients (first and second time derivatives) is shown in Figure 7, achieving an EER of 1.7%. The rest of the paper will use the baseline system for comparison.

## 3.5 User-dependent model orders

Signatures from different users exhibit different degrees of intra-variability: some users sign very consistently, while others alter their signature with each realisation. Thus, it can be expected that using a "system-wide" fixed order for the models will lead to sub-optimal verification performance: the optimal number of Gaussian components will not be the same for all users.

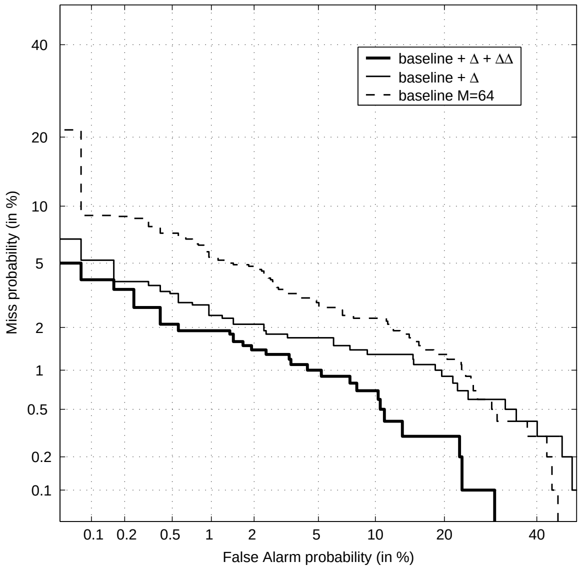


**Figure 7: DET plot for signature verification task with skilled forgeries using Δ and ΔΔ coefficients with the baseline diagonal-covariance matrix 64-Gaussian components GMM**

The procedure proposed here is to train several (full-covariance matrix) models of different orders per user, as well as several world models. Then, given trained model parameters, the MDL criterion can be used to select, for each user, the model which is likely to perform best. The optimal number of Gaussians $\hat{M}$ is selected according to:

$$\hat{M} = \arg\min_{M}(\mathrm{MDL}(\mathbf{\Theta}_M, M)) \tag{11}$$

All other models for that user are discarded, and testing will take place with the selected model. This approach offers the additional benefit of potentially reduced storage requirements and computational costs, because at worst the MDL criterion will select the most complex model for all users (no gain over the fixed-model-order system), but if, as shown in Figure 5, two or more competing models have close average MDL values, lower-order models can be used for some users.

Training several models is expensive, but need only be done once at enrolment. In use, the system stands to benefit from having simpler models to compute the score on. Also, computation of the MDL cost function is inexpensive because in typical implementations of EM the log-likelihood term in Equation 9 can be stored along with the model once each model has been trained, thus needing only a lookup at the MDL cost computation stage.

This algorithm has been tested for selection between 16- and 32-components GMMs trained on 5 signatures each, using the features described in Section 2. Out of 50 users, 23 were assigned a model with 16 components, and 27 were assigned a model with 23 components. The verification performance of the user-dependent model order system compared to the 16- and 32-components system is shown in Figure 8. The user-dependent model order system outperforms the fixed-order system over a wide range of operating conditions, including EER. In comparison to the baseline fixed-order, 64-diagonal covariance matrix Gaussian components system shown in Figure 6, the error rate is smaller at high false alarm probabilities, but higher at EER (about 4.4%).

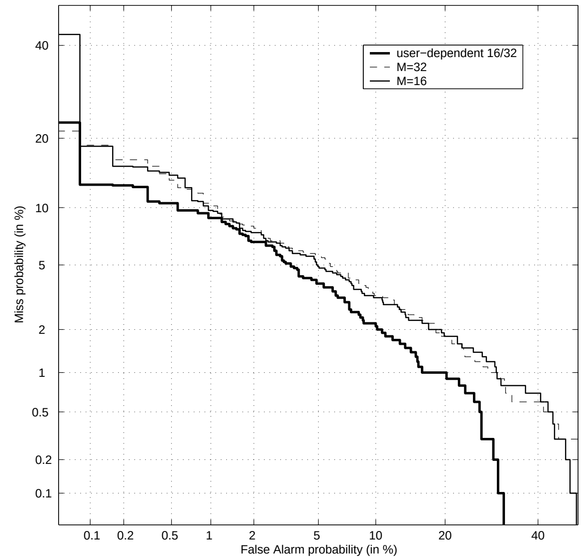


**Figure 8: DET plot for signature verification task comparing user-dependent model order to "system-wide" fixed model orders $M$=16 and $M$=32 in the full covariance matrix case**

## 4. GMMS AND HIDDEN MARKOV MODELS

Hidden Markov models are well suited for the modelling of doubly-stochastic processes, where the behaviour of the features is expected to be time-dependent. At each "time instant", the model instantaneously jumps from a state to another, and observes a feature vector represented by each state's output distribution. The structure of an HMM is described by a number of states $S$, the matrix of transition probabilities between states $\boldsymbol{A}$, and the initial probabilities of each state $\boldsymbol{\pi}$.

The initial probabilities and the transition probabilities are defined as follows:

$$\begin{aligned} &\boldsymbol{\pi}_i = P(s_i) \\ &a_{ij} = P(x(t) = s_j | x(t-1) = s_i) \qquad 1 \le i \le S, 1 \le j \le S \end{aligned} \tag{12}$$

where $x(t)$ is the state at time $t$ and $a_{ij}$ is the probability of making a transition from state $i$ to state $j$. The initial and transition probabilities have to satisfy the constraints:

$$\begin{aligned} &\textstyle\sum_{i=1}^{S} P(s_i) = 1 \\ &\textstyle\sum_{j=1}^{S} P(s_j|s_i) = 1 \quad 1 \le i \le S \end{aligned} \tag{13}$$

For GMMs, both the transition matrix and the initial probabilities vector degenerate to scalars:

$$A_{GMM} = 1 \qquad \pi_{GMM} = 1 \tag{14}$$

Using an HMM implies the assumption that observations are independent given the state. This particular assumption is very likely to be false for signature data, since the feature values, for instance $x$ and $y$, are changing slowly with respect to the sampling frequency. However, the same assumption is made in speech processing and can be partly compensated for by using feature derivatives (delta values).

To fully define an HMM, the parameters of each state's output distribution $b_s(\boldsymbol{o}_t) = P(\boldsymbol{o}_t|x(t) = s)$ need to be specified. The output distributions can be discrete, semi-continuous (SCHMM) or continuous (CDHMM). In signature verification both discrete and continuous distributions are used. Thus, a mixture of Gaussians can be used to model the output distribution for each state of a hidden Markov model: the component weights, means and covariance matrices on the right-hand side of Equation 5 need only be made state-specific with a state index $s$.

As pointed out in [20], the Expectation-Maximisation algorithm formulae for iterative re-estimation of component weights, means and covariance matrices are very similar in the GMM and HMM case, and the usage of EM for training GMM parameters can be seen as a special case of the more general EM for training HMM parameters.

The number of free parameters in an HMM depends on the topology. For a left-to-right topology , the initial probabilities $\boldsymbol{\pi}$ are fixed, so only the transition matrix has to be estimated and stored in addition to each state's output distribution. Thus, for a left-to-right $S$-states HMM with no skips using $M$ diagonal covariance matrix Gaussian components (see Equation 8) to model the distributions in each state, the number of free parameters $N(S, M)$ is

$$N(S, M) = 2(S - 1) + S(M - 1 + 2MD) \qquad (15)$$

Comparisons between the GMM system and the HMM systems are performed trying to keep the number of free parameters in the same range. Since the model parameters are estimated from the same finite amount of training data, this should isolate the effect of topology (as opposed to model order effects) on verification performance.

## 4.1 Model topology and number of states

The most obvious difference between a GMM and an HMM is that HMMs can have more than one state. A crucial decision in designing HMM is to define how many states are needed, and what the possible transitions between them are.

Three broad classes of approaches for discovering the optimal HMM topology and number of states for signature verification exist. The first approach is human expert decision (for instance based on error rates) on a particular topology, where both the transition matrix type and the number of states is fixed. For instance a four-states, left-to-right HMM with skips can be thought to be optimal [21], its performance for the task tested and its topology updated given test results.

The second approach involves fixing the transition matrix type but leaving the number of states free. The data is generally aggressively quantised before applying a structure learning algorithm. Then, discontinuity between quantised feature values can be taken to mean a change of state. This has been used by Muramatsu and Matsumoto [9], where for Japanese signatures pen positions are quantised to 16 directions and the features used are the quantised angles. Imposing a left-to-right topology with no skips, each change of quantised angle in the feature stream creates a new HMM state. Another approach is to make the number of states dependent on the average number of training feature vectors per signature for each user [3].

The third approach leaves both the transition matrix type and the number of states free; a model is learned directly from the observed features. Stolcke and Omohunddro [18] propose a method in which the data is first modelled by a maximum likelihood HMM, which can reproduce the training data exactly, then states are successively merged to obtain a more general HMM using a posterior probability criterion. The reverse approach has been applied in [17], where states are split according to criteria based on goodness-of-fit or the MDL principle. Automated induction of both HMM topology and number of states has to the best of our knowledge not been applied to on-line signature verification problems.

Over the years, researchers have generally reduced the number of states used in HMMs for signature recognition. Thus, while in 1998 and 1999 between 10 and 30 states [7], respectively between 30 and 40 states [2] have been used, more recently in 2002 between 6 and 12 states [5] and in 2003 as little as 2 states [4] have been shown to be effective.

## 4.2 Performance comparison

In keeping with recent research, it was chosen to compare 6- and 2- states HMMs to the GMM baseline system, using diagonal covariance matrices. Two left-to-right topology HMMs with no skips using 2 states with 32 Gaussian components per state (704 free parameters) and 6 states with 11 Gaussian components per state (730 free parameters) were compared with a 64-Gaussian components GMM (703 free parameters). The models were initialised using vector quantisation and a variable number of EM passes to ensure convergence. The test methodology followed is the same as exposed in Section 3.4 for the GMM-based system. The results are shown in Figure 9.

At EER, the performance of the 2-states HMM system is very marginally inferior to the GMM system, while the 6-states HMM system has the same error rate. The difference is significant only for false alarm and miss probabilities below 1%.

Upon examination of the most likely path through the model states given by Viterbi decoding, it was found that for most signatures the training data was split regularly according to the number of states. Thus, the 2-states model splits the data into two clusters, corresponding approximately to the first half of the signature and the second half of the signature. In this case, the time-sensitive nature of HMMs captures some time-dependant specificities, for instance the writing speed is often high at the beginning and/or end of a signature (see Figure 1).

# 5. CONCLUSIONS

This paper introduced the use of Gaussian mixture models for on-line signature verification and provided a performance evaluation which showed them to be effective.

The use of user-dependent model orders based on the MDL criterion was shown to produce significant gains for full-covariance matrix GMMs. Overall, it was found that diagonal-covariance matrix GMMs with more Gaussian components outperformed full-covariance matrix GMMs, at least at EER.

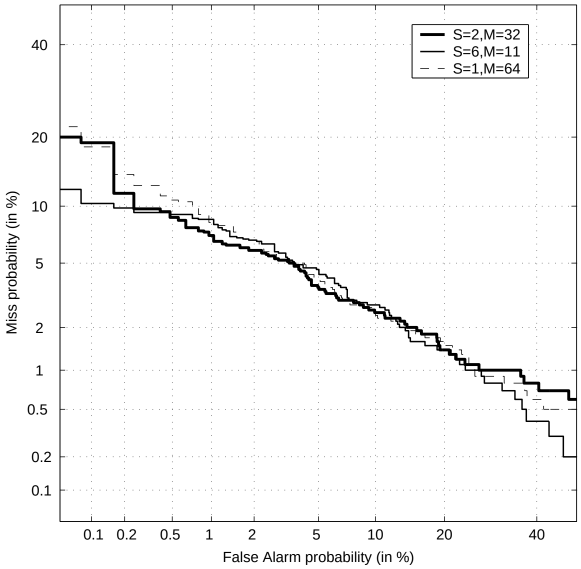


**Figure 9: DET plot for signature verification task with skilled forgeries using 2 states with 32 Gaussian components and 6 states with 11 Gaussian components**

A principled comparison was established between GMMs and HMMs for signature verification applications, which showed that close to EER, the performance of a GMM-based system is remarkably similar to the performance of an HMM-based system of similar order. This indicates that the observed research trend of reducing the number of states in HMMs is justified, and may be an indication that the temporal segmentation effected by left-to-right HMMs is not essential to good discrimination performance.

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